Solving Math Problems with Anime

Sam Spiro

A lower bound on the length of the shortest superpattern

Anonymous 4chan Poster, Robin Houston, Jay Pantone, and Vince Vatter

October 25, 2018

The Haruhi Problem

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What if you wanted to watch the show in all the other 14!-2 ways? Is there an "efficient" way to do this?

The Haruhi Problem

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A	B	Title
01	11	"The Adventures of Mikuru Asahina Episode 00" "*Asahina Mikuru no Bōken Episode 00*" (朝比奈ミクルの冒険 Episode00)
The SOS Brigade previews their movie of questionable quality, directed by Haruhi Suzumiya with nam alien-magician. Mikuru has sworn to protect a young man, played by Itsuki Koizumi, but a love-triangl		
02	01	"The Melancholy of Haruhi Suzumiya Part One" "*Suzumiya Haruhi no Yūutsu I*" (涼宮ハルヒの憂鬱I)
Kyon enters high school as a first year student and meets a strange girl named Haruhi Suzumiya. Du interested in her, Kyon becomes the first person to solicit a normal conversation from Haruhi. This lea		
03	02	"The Melancholy of Haruhi Suzumiya Part Two" "*Suzumiya Haruhi no Yūutsu II*" (涼宮ハルヒの憂鬱II)
Haruhi begins to improve the appearance of the clubroom. Haruhi obtains a computer by staging phot dressing up in a bunny costume and handing out fliers. Later, Yuki invites Kyon to her apartment, whe		
04	07	"The Boredom of Haruhi Suzumiya" "*Suzumiya Haruhi no Taikutsu*" (涼宮ハルヒの退屈)
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```
123               1234123
 231                   2314231
  312                       3124312
    213                           2134213
     132                              1324132
      321                                 3214321
---------         ---------------------------------
123121321   =>    123412314231243121342132413214321
```

Picture from Jeffrey A. Barnett.

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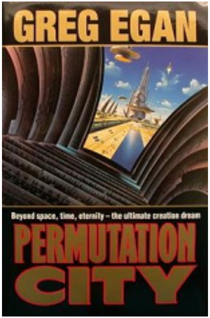

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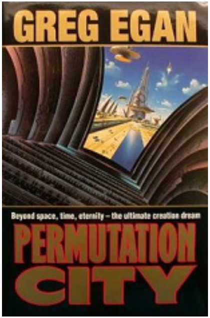
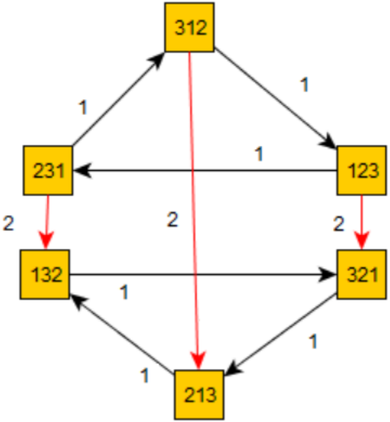


What about lower bounds?

Superpermutations

Construct a weighted digraph as follows. Let your vertex set consist of all permutations on n. Draw an edge between every two permutations where the weight of the edge from π to σ is the minimal number of symbols we need to add to π to get σ. Delete all edges for which the associated transformation produces an intermediate permutation.



Superpermutations

Given an ordered list of permutations $\pi_1, \ldots, \pi_m$ (which we think of as a "walk"), we define $wt(\pi_1, \ldots, \pi_m) = \sum wt(\pi_i, \pi_{i+1})$.

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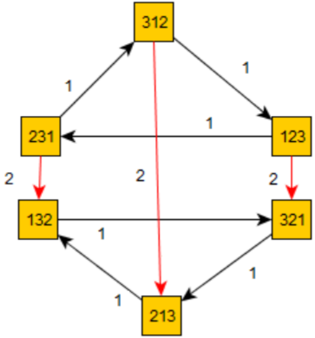
$$wt(\pi_1, \ldots, \pi_m) \geq d(\pi_1, \ldots, \pi_m) - 1 = n! - 1,$$

since we assumed the walk of π visits every permutation.

Superpermutations

Define the 1-loop of a permutation π to be the set of permutations that π can reach by only using edges of weight 1. Observe that the number of 1-loops is $(n-1)!$.



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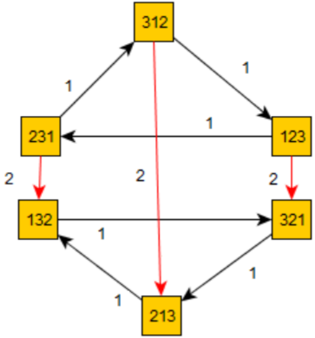

Define $c(\pi_1, \ldots, \pi_m)$ to be the number of 1-loops that the walk $\pi_1, \ldots, \pi_{\mathbf{m-1}}$ has completely gone through (note the index of that last step of the walk!).

Superpermutations

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If $wt(\pi_{m-1}, \pi_m) \geq 2$ then the lefthand side increases by at least 2, but the righthand side increases by at most 2 (for every step of the walk), so the inequality holds.

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If $wt(\pi_{m-1}, \pi_m) = 1$ then the walk didn't leave its 1-loop, so either (1) it didn't visit a new permutation or (2) it didn't finish a 1-loop. In either case the righthand side increases by at most 1. We conclude the result.

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Corollary (Ashlock and Tillotson, 1993)

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This was all that was known by the combinatorics community. However, while working on the Haruhi problem, someone on 4chan managed to improve this bound!

Superpermutations

Observe that there is a unique edge from π of weight 2, i.e. the one which goes to $\pi(3)\cdots\pi(n)\pi(2)\pi(1)$. E.g. 51234 goes to 23415.

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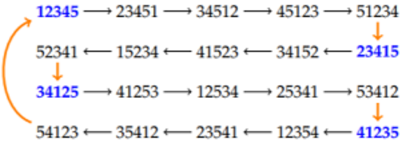
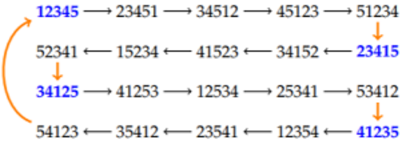
The 2-loop generated by π is defined as the set of vertices visited by the walk that starts at π, follows $n-1$ consecutive edges of weight 1, then follows the (unique) edge of weight 2, and then repeats these steps $n-2$ more times.



Picture from "A Lower Bound on the Length of the Shortest Superpattern."

Superpermutations



Observe that this 2-loop is generated precisely by all of the bold permutations in the above picture (i.e. by fixing the last entry of 12345 and then cyclically generating the elements).

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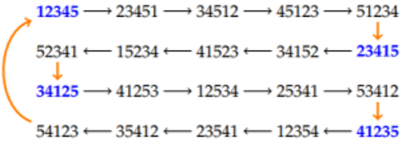

Observe that this 2-loop is generated precisely by all of the bold permutations in the above picture (i.e. by fixing the last entry of 12345 and then cyclically generating the elements). Also observe that each 2-loop contains exactly $n(n-1)$ elements.

Superpermutations

We say that a walk visits the 2-loop generated by π if it follows an edge of weight 2 or more to arrive at π. Note that this means that the 2-loop we are at depends not only on the vertex we are currently at, but also how we got there.

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Let $t(\pi_1, \ldots, \pi_m)$ denote the number of 2-loops visited by the walk where we let $t(\pi_1) = 1$. Note that since each 2-loop contains $n(n-1)$ permutations, a walk visiting every permutation must enter at least $(n-2)!$ different 2-loops.

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Superpermutations

Assume $wt(\pi_{m-1}, \pi_m) = 2$. We claim that c and t can't both increase, which will give us the result.

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$$\geq n! + (n-1)! - 1 + (n-2)! - 2,$$

so we conclude the result.

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Theorem (Janson; February 23 2024)

The advantage is asymptotic to

$$\frac{3\sqrt{3}}{2\sqrt{\pi}}\sqrt{n}.$$

More General Games

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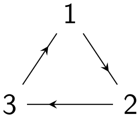
Given a digraph D, define the D-game by having two players simultaneously pick vertices of D each round.



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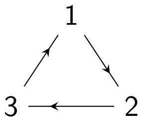

Given a non-negative integer vector $\vec{r}$, the *semi-restricted* D-game (with parameter $\vec{r}$) is defined by having players Rei and Norman iteratively play the D-game, with the restriction that Rei must play vertex v exactly $\vec{r}_v$ times.

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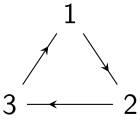

Given a non-negative integer vector $\vec{r}$, the *semi-restricted* D-game (with parameter $\vec{r}$) is defined by having players Rei and Norman iteratively play the D-game, with the restriction that Rei must play vertex v exactly $\vec{r}_v$ times. E.g. if D is as above and $\vec{r} = (n, n, n)$, then this is semi-restricted RPS.

Optimal Scores

Let $S_D(\vec{r})$ be the expected score for Norman in the semi-restricted D game is both players play optimally.

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Optimal Strategies

Theorem (S.-Surya-Zeng; 2022)

If D is the directed path $1 \to 2 \to 3$, then a strategy for Rei is optimal if and only if she plays 3 with probability $1/2$ whenever she can.

$$\begin{array}{ccccc} \mathbf{1} & \longrightarrow & \mathbf{2} & \longrightarrow & \mathbf{3} \\ p & & 1/2 - p & & 1/2 \end{array}$$

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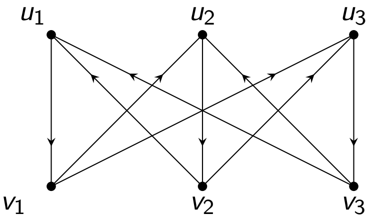
Question

Does every digraph D have an optimal strategy for Rei which is "oblivious", i.e. which only looks at which u Rei can play and ignores how many times she can play it?

Optimal Strategies

Theorem (S.-Surya-Zeng; 2022)

The digraph depicted below does not have an oblivious optimal strategy for Rei.



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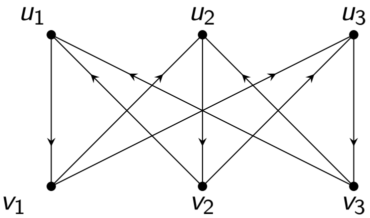

Theorem (S.-Surya-Zeng; 2022)

Almost every Eulerian tournament does not have an oblivious optimal strategy for Rei.

Proofs: Bounds

Theorem

$$S_D(n, \ldots, n) \leq \max_v (d^+(v) - d^-(v))n + C_D n^{1/2}.$$

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Consider the following strategy for Rei: uniformly at random pick $v \in V(D)$ until some option runs out, then play arbitrarily. Until something runs out, Norman can gain at most

$$\max_v \frac{d^+(v)}{|V(D)|} - \frac{d^-(v)}{|V(D)|}$$

points in expectation each round.

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One can show that in expectation only $C_D n^{1/2}$ turns remain after Rei runs out of some vertex to play. □

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$$S_D(\vec{r}) \leq \max_v \sum_{u \in N^+(v)} \vec{r}_u - \sum_{u \in N^-(v)} \vec{r}_u + C_D M^{2/3},$$

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After something runs out, we expect the number of actions for any v to be at most $\vec{r}_v^{\,-1/2} \sum_u \vec{r}_u$

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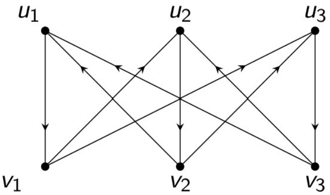
Proofs: Strategies

Lemma

For RPS we have $S_D(\vec{r} - \delta_s) \leq S_D(\vec{r} - \delta_p) + 1$.

Proofs: Strategies



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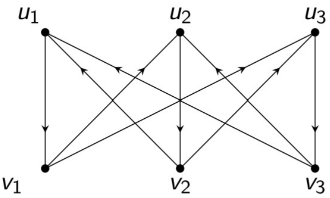

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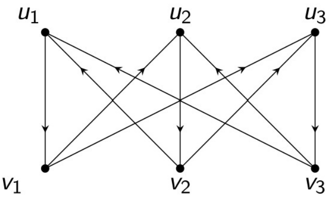

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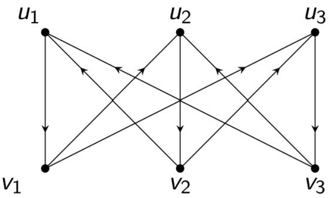

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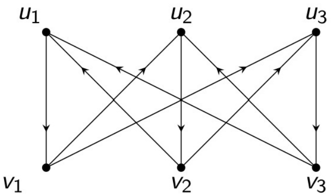

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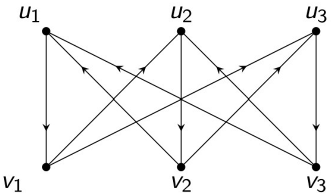
Open Problems

Question

What are the optimal strategies for the semi-restricted D-game with D as below?



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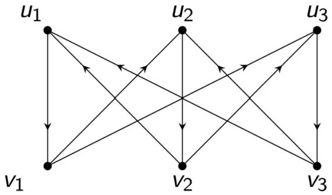

Question

What are the optimal strategies for directed paths?

あなたは多分日本語が読めません！